Critisize Atomism : Hystorical and Metaphysical Investigation of Quantum Mechanics

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Introduction

Let us assume a system of two helium atoms. Suppose that they were observed at time t at points A and B respectively, and then one atom was observed at point C at time $t + \Delta t$. You should mind that you can set up the system with no restriction so that $\overline{\mathrm{AC}}/\Delta t$ will be larger than light velocity and $\overline{\mathrm{BC}}/\Delta t$ will be smaller.

However, if you accept the principle that you can not distinguish atoms of the same kind, you can not deny the possibility that the atom of point A has moved over light velocity. As long as you do, speed of atom can easily exceed light speed. That is, the existence of atom doesn't agree with the theory of relativity.

The quantum mechanics is right. The theory of relativity is right. Yet atomism is not right. We have to pursue physics without the concept of atom.

Part I
Electron

1

1.1

The concept of electron is obscure. How is the size of electron? What is spin of electron?

Spin of electron must really exist because we can actually observe magnetization of matter. If electron is the source of magnetism and has magnetic moment itself, it must have finite size because magnetic moment has direction but single point can not. That is, electron must contain at least two points in order to have magnetic moment.

In conclusion, electron does exist and have finite size.

1.2

Then, how can we decide the size of electron?

Size of object is in principle defined by contact with others because contact surface forms its boundary. It is the surface that determines its size. However, contact between objects results from Coulomb force and Pauli principle. If size of object is defined by Coulomb force, how can we define size of electron which is the source of Coulomb force?

1.3

We shall now investigate in detail the source of mechanical force between objects in contact so as to answer the above question.

To take an example, let us consider two ionic crystals in contact with each other so lightly as not to change their forms. Van der Waals force will operate on their boundary surfaces in this case. If the force results from Coulomb force, it is clear that Coulomb force determines the size of object.

If you examine from another point of view, you will be aware of the following matter. Since the ions on surface don't change their positions, there must be reactions on them from the inside in the same strength as on the surface. This reaction results from the bonding force of the crystal. Coulomb force among the ions in crystal maintains its structure. Therefore Coulomb force keeps the form of object in this case.

Nevertheless, Coulomb force is not the only factor. We have to consider exchange force together.

1.4

Magnesium neucleus consists of twelve protons and of the same number of neutrons, for example. Magnesium atom also has twelve electrons, so it is boson as a whole. Therefore the exclusion principle must not operate on magnesium atom, but in fact solid magnesium possesses finite size. Why?

It is because electron of one atom repels that of another. Then, is it Coulomb force or exchange force that works among atoms and maintains stucture of matter?

Since magnesium atom itself has no charge, Coulomb force doesn't operate even if two atoms are in close. Yet in a certain distance, they start to polarize and attractive force results from their dipole moments. If you bring them closer, electrons spreading on the outermost of them bigin to contact together. Then, if two atoms were carbons, orbits of two electrons would merge and covalent bond would come up. That is not the case for magnesium because energy is not minimized with covalent bond. Instead, repulsive force is caused by clossing of orbits of electrons.

1.5

What is exchange force?

It is the force operating between two fermions based on Pauli principle which forbids them in the same state. Since electron is fermion, the repulsive force operates on it. Even if you bring two electrons close against repulsion, it is forbidden by the principle for them to be in the same state. That is, they can not be superimposed.

However, you have to pay attention to the following matter. If a fermion doesn't have electric field, exchange force will actually not work on it because it is a charge of fermion that makes discrete energy levels formed under electromagnetic potential, which causes exchange force. That is, although exclusive principle is in effect irrespective of electric property, exchange force itself results from charge of fermion.

Also, the principle must work even on boson as long as it is composed of several fermions.

The above argument clarifies that both exclusive principle and Coulomb force sustain structure of matter. We can also say that the two principles give grounds for contact between objects necessary to decide size, and mainly work on electron.

Then, if electron makes contact possible, what does it contact with?

1.6

What is electron in the first place?

It is the source of electric field and also is matter affected by the field.

To take an example, let us consider a spherical conductor with negative charge. When the sphere is under electric field, force is applied on it according to field intensity. Then, what does the force work on?

It works on electrons within the sphere. Since the sphere and electrons are tightly bound together, forces on electrons affect the whole sphere.

Electron becomes free from binding and springs out of sphere when the field increases over a certain intensity. It keeps moving along the field until it hits another object, which is called cathode lay. Electron seems moving in this case.

Yet it is well-known that we cannot describe motion of electron. We can know where it was in the beginning and where it arrived in the end, but we cannot know where it passed.

If we cannot, why can we assert it moves? Is it really moving? Although we have not defined size of electron, how should we think about its motion?

2

2.1

We shall now review the problem of electron in the classical mechanics and electromagnetics [1].

In round terms, there were two opinions about size of electron. The first is that electron has no volume and is a geometrical point. The second is that electron is a sphere with finite size and its charge is uniformly distributed on the surface. Each theory offers difficulty.

2.2

Now we shall examine the former. The first difficulty is that self-energy of electron would have an infinite value.

Let us assume that there are two point charges of the same value $-q$ apart at a distance r at rest. The electrostatic energy stored between them will be proportional to q^2/r. Therefore the energy infinitely increases as the distance r approaches zero. If you regard electron as a point, you will need an infinite energy to collect finite charge of $-e$ into one point.

Nevertheless, some people say that the infinite self-energy doesn't matter at all. The idea that a point charge interacts with itself is nonsense.

The second difficulty is radiant resistance. Generally speaking, a charged object radiates electromagnetic waves when it moves in acceleration. Since the wave has momentum, reaction will work on the object.

The problem is what generates this reaction in the case of electron. If the charged object has a finite size, you can suppose that the interaction between two parts of the object produces action and reaction. However, point charge has no portion. Even if there is only one electron in a vacuum, reaction must work on it. Therefore we have no choice but to consider that electron interacts with itself. If we do, we cannot discount the problem of self-energy.

2.3

Then, let us consider the latter theory that electron is a sphere of finite size. There was a problem how electric charge is distributed, and Poincaré supposed that it is uniformly distributed on the surface of sphere. He then introduced the special force for charges repelling each other to be bound on the surface, which is called Poincaré stress.

Nevertheless, we have to think about the question whether the small charges on surface have volumes.

If they do not, there is the same difficulty as of the former theory.

If they do, how is the shape and distribution of the charges? That is, the same problem can be raised about small charges on the spherical surface as about electron. Therefore this theory offers difficulty of infinite regress.

2.4

The above two theories are genuinely classical. We shall now examine the quantum mechanics for size of electron.

However, the question is hardly introduced there because we have no problem to describe the mechanics if we know only the observable mass and charge of electron.

In the quantum mechanics, electron is described having both features of wave and particle. A wave function represents electron as a probability wave at a certain time. When someone observes electron, the wave function collapses and electron appears as a particle at one point on space.

Then, how can we decide the size of electron when it appears as a particle? Although we cannot define its size, is it proper to call it particle?

In the experiment of double slit, for example, each electron will mark small spot on screen. The area of spot is certainly almost zero compared with the spread of wave function, but it is not confirmed yet that electron is really a point without volume.

The problem is not solved even in the quantum mechanics.

3

3.1

Is electron matter in the first place?

If matter has to possess volume, electron is not matter because we can not define its size.

According to the uncertainty principle, you cannot measure both momentum and position of electron at the same time with an arbitrary accuracy. You can estimate the size of hydrogen atom by applying this principle to electron around hydrogen nucleus. The size of hydrogen atom is that of electron cloud around nucleus.

Electron moves around hydrogen nucleus. Since nucleus is positive and electron is negative, there will be Coulomb attraction between them. However, electron does not fall into nucleus. Why not?

It is because electron keeps moving in direction perpendicular to the attraction with fixed momentum. It is just like the motion of the earth around the sun. If the earth stopped, it would fall into the sun, so that the earth must keep moving not to fall. Also, there is a relation between the momentum of the planet and its orbital radius. Therefore the orbital radius of electron is determined by its momentum, which is the size of atom.

Nevertheless, every electron does not necessarily have the same momentum, doesn't it?

If electrons of several atoms have different momentums, sizes of hydrogen atoms will be varied. Yet atom size has to be uniform because the spectrum of hydrogen atom is always of the fixed frequency and the frequency is related to orbital radius of electron. This very consideration led us to the uncertainty relation, which was needed for electron orbit to be uniform.

However, charged object must radiate electromagnetic waves in rotational motion. Why doesn't electron around nucleus radiate waves?

It may be because nothing moves within hydrogen atom. We may not be able to describe motion of electron within atom.

In concusion, it is clear that electron is not matter. It is rather a property of matter.

3.2

It is well-known that electron shifts its position. It moves freely in metal or as cathode lay and tunnels between two atoms.

It is when electric field is applied and when charged object moves that electron moves. The surprising thing is that we can distinguish motion of object from motion of electric current within it. We usually explain this fact by the idea that carrier of current is different from the matter through which current flows. Current results from the motion of carrier within matter. Then, how is the size of that carrier?

If electron itself is not matter but just a property of matter, its size must be equal to that of object it belongs to. If it is, there are two kinds of motion on matter, shift in position and motion of current inside. Mass of electron can be understood as inertia of current.

3.3

Current also interacts with external field, which enables radiation. However, is it possible that one current element interacts with another in the same object?

If each element of current is spatially localized, interaction between them will be possible. Yet we do not know how to devide current. If current is a substance and exists over the whole body of conductor, is it allowable to devide that?

This is the problem that comes up when you regard electron as the same size as conductor. Can we count it?

It may be that electrons can not interact with each other within conductor and they interact only with external field. There is Coulomb interaction between charged bodies but may not be between electrons. This difference is important when you think about electrons within the same object. Coulomb interaction is in effect only between electrons localized in limited domains but is not between electrons spread over the whole matter.

This argument lets us know the importance of localization. It is only when electrons are localized that they interact together. However, the condition of localization is not clear. If localization results from Coulomb interaction, our explanation would be reversed.

This argument also implies that the single-particle approximation of conduction electron is not an approximation.

4

Assume that there is one electron in a vacuum. If there is no external field, the space must be isotropic. Then, what direction is the magnetic moment of electron pointing to?

Electric field accompanying electron is isotropic but the field of magnetic dipole is anisotropic. Therefore the dipole must point to fixed direction because there is no reason for rotation. What direction should that be?

We cannot determine such direction. The field of isolated electron has to be isotropic. Therefore I propose that the most basic state of electron is spin-singlet state composed of two electrons. This state can be completely isotropic, provided that the state is of zero magnetic field, neither magnetic dipole nor quadrupole.

This pair of electrons doesn't separate into two until magnetic field is applied. Then, two electrons begin to possess magnetic moments opposite to each other along direction of external field. This idea gives us the better understanding of the experiment of Stern-Gerlach.

If there were electrons with magnetic moments of arbitrary directions from the beginning, continuous figure would appear as a result of the experiment. However, if magnetic moments of electrons don't appear until external field is applied, we can explain that there are only two lines corresponding to opposite magnetic moments.

Although we can explain neither how magnetic moment appears when external field starts applied nor why it has finite magnitude from the beginning, but this hypothesis has a physical necessity led by two different facts, spatial isotropy and magnetization of matter.

Part II
Photon

5

5.1

The energy density of black body radiation is described as

$$u_\nu d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}\,d\nu \tag{1}$$

in the Planck's formula [2]. According to Fourier expansion of black body radiation

$$E_z = \sum_{n=0}^{\infty} C_n \cos\left(2\pi n\frac{t}{\tau}-\theta_n\right), \tag{2}$$

where τ is a basic period, and with a formula of electromagnetism

$$u = \frac{3}{4\pi}\overline{E_z^2}, \tag{3}$$

we obtain another formula of the energy density

$$u_\nu d\nu = \frac{3}{8\pi}\sum_{n}^{n+\Delta n} C_n^2. \tag{4}$$

Δn is a number of radiations whose frequencies are between ν and $\nu + d\nu$, so that $d\nu = \Delta n/\tau$. The mean value of square of radiation intensities between n and $n+\Delta n$ can be described as

$$\overline{C_n^2} = \frac{1}{\Delta n}\sum_{n}^{n+\Delta n} C_n^2 \tag{5}$$

and consequently

$$u_\nu d\nu = \frac{3}{8\pi}\tau\,\overline{C_n^2}\,d\nu. \tag{6}$$

From (1) and (6), we obtain

$$\overline{C_n^2} = \frac{64\pi^2 h\nu^3}{3\tau c^3}\frac{1}{e^{h\nu/kT}-1}, \tag{7}$$

which goes to

$$\overline{C_n^2} = \frac{64\pi^2 h\nu^3}{3\tau c^3}\,e^{-h\nu/kT} \tag{8}$$

in the limit of high frequency.

We can interpret this formula as a restriction on Fourier components of black body radiation. The intensity of Fourier component C_n must decrease with the rate proportional to $e^{-h\nu/2kT}$ as frequency becomes higher so as to reproduce the spectrum of black body radiation.

In classical physics, there is no reason to put such restriction on Fourier components, so that physicists postulated the infinite number of high frequency components within limited space. This assumption resulted in the Rayleigh-Jeans law, which did not describe all features of the spectrum.

However, is it possible that limited space contains infinite number of electromagnetic waves?

If there can be only limited amount of substance within limited space, the amount of electromagnetic waves must be limited too. We will regard an electromagnetic wave of each frequency as substance in this point of view, which can be justified if we assume that electromagnetic wave has energy and momentum by itself. This viewpoint leads us to the concept of continuum as Aristotle [3] and Kant [4] argued once.

Since there cannot be infinite number of electromagnetic waves within limited space and there is no assumption in advance about the number of waves around arbitrary frequency in thermal radiation, the proposition is reasonable that the probability of wave decreases as frequency increases. We do not need to introduce a concept of photon in this explanation.

We can choose another way to cut off waves over a certain frequency. If we do, however, we can not uniquely determine the cut-off frequency.

5.2

Nevertheless, this solution leads us to another problem. There is no scale of length in formula (7), that is, the probability of each frequency component doesn't change even if we double the volume of cavity filled with radiation. That simply depends on temperature.

According to our assumption that there is only limited amount of substance within limited space, the number of waves must increase if the volume of cavity is doubled, but that is not the case. The amount of electromagnetic substance is determined not by volume but by temperature.

Since the radiation formula (2) originally represents components of electric field at one point in space, the formula obviously doesn't depend on spatial scale. Therefore the entire energy of radiation increases in proportion to volume.

Planck's formula is a pre-spatial expression because that doesn't contain spatial variable and consequently is able to give natural unit of length. This fact implies that radiation is essentially not related to space.

5.3

The derivation of Planck's formula depends on hypothetical oscillators radiating and absorbing electromagnetic waves. That is, it was entropy of oscillators that Planck actually estimated and he could not directly calculate entropy of radiation [5].

To take an example, we shall try to consider a width of frequency of photon with frequency ν. What determines width of specral line?

It is the probability of transition corresponding to emission of photon with the uncertainty principle. Then, is there any relation between entropy and width of frequency of photon?

Monochromatic light can be described as superposition of lights of various frequencies within some range. This rule also applies to photon because every spectral line is somewhat broad. If it does, you have to suppose that possible combination of frequencies composing a photon will be related to its entropy as long as you regard entropy as logarithm of number of microstates. However, you can not derive Planck's formula in this way no matter how cleverly you try. If you doubt, you should try to calculate by yourself.

If monochromatic light carried entropy, it should be entirely different from that in the theory of Boltzmann because we can not calculate number of microstates of radiation so as to reproduce Planck's formula.

This argument leads us to the conclusion that properties of light are determined not by light itself but by properties of matter related to radiation. If you agree to this, it is natural to think about whether there is any necessity to regard light as reality although all properties of light are determined by matter. Instead, you should regard transport of energy and entropy as direct relation between matters without supposition of light.

5.4

When you think about light, you will notice that what you observe is not light itself but color of object. Photon gas is supposed to have energy on blackbody radiation, but it is properties of radiant matter that you actually observe and you can not measure properties of light itself.

It is energy decrease of object A and increase of object B that you can observe. If you put some observation equipment between A and B, you should replace B with the equipment and name B'. Then the same argument applies to A and B'.

The energy conservation law holds by the assumption of light. However, if you consider that energy conservation means causality, you can regard the time points of energy decrease on A and of increase on B as the same time. Then, is there any reason to postulate real existence of light?

This viewpoint agrees with the theory of relativity because events A and B are at the same time for light itself in the theory. You can understand relation between the quantum theory and the theory of relativity for the first time when you abandon existence of light.

6

6.1

Although atom was introduced as a postulate on statistical mechanics, results of theory of that kind didn't agree with several experiments (for example, specific heat of diatomic molecule gas, of solid matter and black body radiation). Then the quantum mechanics had solved the problem, so that consequences of statistical mechanics became consistent with experiments and it was recognized as a scientific theory.

However, was the original postulate really survived during that process? It was rather by abandonment of atomism that the theory came to agree with experiments, wasn't it?

What does the factor $(N!)^{-1}$ of partition function mean?

Why is the product of Boltzmann constant and Avogadro constant equal to gas constant? Einstein seemed puzzled about this fact [6].

And what does Planck constant mean in the first place?

We surely archieved the agreement of statistical mechanics and variety of experiments owing to the invention of quantum mechanics, but all of these was done by elimination of atom. Then, hardly any one would be aware of the implication of that theory.

6.2

The closest thought of quantum mechanics is not of Democritus but of Eleatics of denial of motion because we can not describe motion by the theory.

What is described in the mechanics is leap from one state to another. It is just like a jump between two photographs. It shows us only probability of leap between two states, a transition probability.

If we take an example of Zeno's paradox of flying arrow, what we can describe through the quantum mechanics is only the probability of transition from one state of the arrow at one moment to another state at another moment. That describes the arrow as being stopped all the time and claims it is meaningless to think about the process of flying.

It only confirms the paradox but doesn't solve. The arrow is still stopped even in the quantum mechanics. We have to describe motion and how an arrow flies.

The amount of substance within limited space and time must be limited. We once tried to explain it from the concept of atom, which has already turned out inappropriate. We have to search for proper representation of finite continuum. It is not the cheap trick of complementarity but the understandable explanation that we need.

References

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